

Pricing with Rough Bergomi Model in Commodity Markets

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Abstract: In this work, we present an analysis of the applicability of rough volatility in commodity markets. We propose a rough volatility model for pricing European options on commodity futures, based on the work of Nastasi et al. (2020). Furthermore, we present an efficient numerical scheme to simulate the model and calibrate it to real market data on WTI Crude Oil.

1 Introduction

In Comte and Renault (1998), the authors proposed the Fractional Stochastic Volatility (FSV) model as an extension to the Hull-White model, by replacing the driving Brownian motion with a fractional Brownian motion (fBm). This process generalizes the classical Brownian motion and depends on the so-called Hurst parameter $H \in (0, 1)$, which governs the smoothness of the trajectories. When $H = 0.5$, the classical Brownian motion is recovered; for $H < 0.5$, the trajectories become rougher, whereas for $H > 0.5$ they are smoother.

In the FSV model, a value $H > 0.5$ was assumed as this was consistent with the prevailing perception that the volatility of asset prices exhibited a long memory. Several years later, in Gatheral et al. (2018), this assumption was revisited. The authors empirically studied the realised volatility of several securities and verified this assumption, for reasonable timescales of practical interest. More precisely, the log-increments of the realised volatility seemed normally distributed and enjoyed a scaling property with constant parameter, that is, they were H -self-similar for some constant $H > 0$. This analysis also suggested that the ‘smoothness parameter’ H was in fact rather low, between 0.06 and 0.2, thus implying a rough volatility.

Since Gatheral et al. (2018), rough volatility models have attracted significant attention, as they are able to reproduce market dynamics more accurately than classical stochastic volatility models, such as Bergomi or Heston models. However, most of this attention has been focused on applications within equity markets, while other financial markets, such as commodities, remain relatively unexplored.

In this work, our aim is to bridge this gap by presenting a novel analysis on the applicability of rough volatility in commodity markets. First, in Section 2 we outline the main characteristics of derivatives pricing in commodity markets. Next, in Section 3 we introduce a novel model to price European options on commodity futures, which incorporates rough volatility through the rough Bergomi model (rBergomi) proposed in Bayer et al. (2016) for equity markets. Next, in Section 4 we discuss its simulation and calibration, and finally we calibrate the model to market data on WTI Crude Oil and analyse the results in Section 5.

2 Options on Futures in Commodity Markets

Commodity markets are financial markets where raw materials and other physical goods are traded, such as crops, gold, and crude oil. The main characteristic of commodities is that they are physical objects that need to be stored and transported, thus involving logistical challenges when trading them. Derivative contracts allow traders to participate in the market without dealing with any of these issues. Due to this, futures contracts are the most liquid commodity contracts, followed by options on futures.

Due to the physical nature of commodities, futures contracts customarily stipulate a period in which the seller can deliver the goods at any time. We denote that period as $[T_0^d, T_1^d]$, where T_0^d and T_1^d are the start and final delivery dates, respectively.

The seller must notify the buyer a few days prior to the delivery. Since the delivery can happen at any time during $[T_0^d, T_1^d]$, the notification can also happen during a certain stipulated period $[T_0^n, T_1^n]$, where T_0^n and T_1^n are the first and last notification dates, respectively. The only restriction is that the notification must occur before the delivery. Futures option contracts can be traded during $[T_0^o, T_1^o]$ and this is always well before T_1^f and T_0^n , where $[T_0^f, T_1^f]$ is the period during which the underlying futures contract can be traded.

We denote by S_t the spot price of the commodity at time t and by $F_t(T)$ the price at time t of a futures contract with maturity date T . We omit T in notation $F_t(T)$ when unambiguous.

In equity markets, it is well known that under standard assumptions on absence of arbitrage, the future price converges to the underlying spot price at delivery and an equivalent measure exists such that the futures price process is a martingale. In commodity markets, that is not true due to the delivery options: in general, commodities spot prices may not coincide with futures prices at maturity, so we cannot apply the same arguments as in equity markets.

However, if we consider the availability of futures contracts as hedging instruments, we can find an equivalent measure such that we can assume the existence of a single time-continuous process S_t , called the **fictitious spot price**, such that

$$F_t = \mathbb{E}_t[S_T], \quad t \leq T, \quad (5.1)$$

for any futures contract on the same underlying.

3 Commodity Market Modelling

Based on the model presented in Nastasi et al. (2020), we propose a linear stochastic volatility model for the fictitious spot price S_t in which the variance v_t is given by the following rBergomi model:

$$\begin{cases} dS_t = (\alpha(t) + \beta(t)S_t) dt + \sqrt{v_t}S_t dW_t^1, & S_0 \text{ given} \\ v_t = \xi_0(t) \exp\left(\frac{-\eta^2}{2}t^{2H} + \eta\sqrt{2H} \int_0^t \frac{dW_s^2}{(t-s)^{0.5-H}}\right), \end{cases}$$

where $\xi_0(t)$ is the initial forward variance curve, η is a positive constant, $H \in (0, 1)$ is the Hurst parameter, $\alpha(t)$ is a positive function of time, $\beta(t)$ is a function of time and W^1, W^2 are Brownian motions with correlation $\rho \in [-1, 1]$.

For convenience, we introduce the **normalised fictitious spot price**:

$$s_t := \frac{S_t}{F_0(t)},$$

the dynamics of which follows from the product rule and satisfies the stochastic differential equation:

$$ds_t = (a(t) + (\beta(t) - \partial_t \ln F_0(t))s_t) dt + \sqrt{v_t}s_t dW_t^1, \quad s_0 = 1,$$

where

$$a(t) := \frac{\alpha(t)}{F_0(t)}.$$

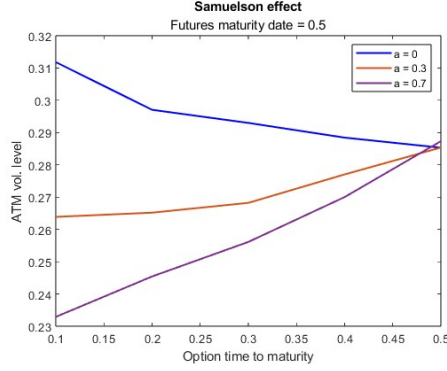


Figure 1: ATM implied volatility with respect to the options maturity date of a mid-curve option and for different mean reversion speeds. The Samuelson effect is more pronounced as the mean reversion speed increases.

We can then impose Equation (5.1) to establish the relation between the normalised fictitious spot price s_t and the futures prices F_t , and by doing so we will automatically calibrate the functional parameter $\beta(t)$ and obtain the following final model:

$$\begin{cases} ds_t = a(t) \cdot (1 - s_t) dt + \sqrt{v_t} s_t dW_t^1, & s_0 = 1, \\ v_t = \zeta_0(t) \exp\left(\frac{-\eta^2}{2} t^{2H} + \eta \sqrt{2H} \int_0^t \frac{dW_s^2}{(t-s)^{0.5-H}}\right), \\ F_t(T^f) = F_0(T^f) \cdot \left(1 - (1 - s_t) e^{-\int_t^{T^f} a(u) du}\right), \\ dW_t^1 dW_t^2 = \rho dt, \end{cases} \quad (5.2)$$

This derivation is non-trivial and required the development of new theoretical results to justify this model. A detailed derivation is provided in Folgar-Cameán (2025).

We observe that the model (5.2) exhibits a mean reversion phenomenon with mean reversion speed $a(t) > 0$. This is consistent with the Samuelson effect: a stylised fact of many commodity markets (such as crops, natural gas and crude oil), which states that commodities futures prices become more volatile when approaching the maturity date. Several authors assert that the Samuelson effect is closely related to mean reversion behaviour of the spot price, see for instance Bessembinder et al. (1996). In Figure 1 we observe how the ATM implied volatility level of mid-curve options predicted by the model increases when approaching the maturity date for positive mean reversion speeds, which is in line with the Samuelson effect.

4 Simulation and Calibration of the Model

The dynamics of the spot price s_t can be simulated using the Euler-Maruyama scheme. However, the simulation of the variance process is not as direct due to the presence of the fractional Brownian motion, whose simulation is an active area of research (see Matas and Pospíšil (2022), for example).

In this work, we use the Hybrid Scheme proposed in Bennedsen et al. (2017) to simulate the fBm efficiently, which we implemented on MATLAB R2024b.

For computational efficiency, we assume a constant mean reversion speed a from now on, unless explicitly stated otherwise.

We calibrate the model (5.2) to market data of vanilla options following a nested calibration

procedure. In the outer loop we calibrate η, ρ and H by minimizing the cost function:

$$L(H, \rho, \eta) = \sum_i \frac{\omega_i}{\omega} \left| \sigma_i^{mkt} - \sigma_i^{rB} \right|, \quad (5.3)$$

$$\omega_i := \frac{vol_i}{\max(0.01, BA_i)}, \quad \omega := \sum_i \omega_i,$$

where the index i cycles through all the contracts considered for the calibration, vol_i is the trading volume of the i -th contract, BA_i is its bid-ask spread, σ_i^{mkt} denotes its quoted implied volatility and σ_i^{rB} the implied volatility calculated from the prices generated by the model. We solve the optimization problem defined by the cost function (5.3) with the surrogate model optimizer `surrogateopt` provided by the Global Optimization Toolbox in MATLAB 2024b.

For computational efficiency, we will assume that the forward variance curve $\xi_0(t)$ is given by a piecewise constant function with levels $c_i \in [0, 1]$ and discontinuous at the maturity dates of the contracts we are calibrating the model with. For each evaluation of the cost function we calibrate the levels $\{c_i\}$ sequentially by the bisection method. We summarise the procedure in Algorithm 1.

Note that we are not calibrating the mean reversion speed a since it cannot be robustly calibrated to vanilla options alone: indeed, we can successfully calibrate the model for any fixed value of a between 0.3 and 0.7. This could be addressed by enriching the calibration set with options more sensitive to this parameter, such as mid-curve and calendar spread options. However, since we do not have access to market data for these instruments, we fix the mean reversion speed to $a = 0.5$, which our experiments confirm to be a reasonable choice.

Algorithm 1 Nested calibration procedure

```

1: procedure CALIBRATEMODEL
2:   Initialize  $(H, \rho, \eta)$ 
3:   while not converged do
4:      $L \leftarrow 0$ 
5:     for each contract  $i$  do
6:        $c_i \leftarrow \text{BISECTION}(i, H, \rho, \eta)$ 
7:        $\sigma_i^{rB} \leftarrow \text{MODELIMPLIEDVOL}(i, H, \rho, \eta, \{c_j\})$ 
8:        $\omega_i \leftarrow vol_i / \max(0.01, BA_i)$ 
9:     end for
10:     $L \leftarrow \sum (\omega_i / \omega) |\sigma_i^{mkt} - \sigma_i^{rB}|$ 
11:    Update  $(H, \rho, \eta)$  to minimize  $L$ 
12:  end while
13:  return  $(H, \rho, \eta, \{c_i\})$ 
14: end procedure

```

5 Results

We calibrate the model to market data for OTM vanilla futures options on WTI Crude Oil, traded on NYMEX (New York Mercantile Exchange) on the 14th of March of 2025. We consider options with 6 different maturity dates T_1^o on futures contracts with 6 corresponding maturity dates T_1^f , as indicated in Table 1. We calibrate the model to 166 distinct contracts; 46015 when weighted by trading volume.

The obtained calibrated parameters are shown in Table 2. In Figure 2, for six different dates, we can observe the volatility smiles generated by the model and compare them to the market

T_1^o	T_1^f
16th of April 2025	22nd of April 2025
15th of May 2025	20th of May 2025
16th of June 2025	20th of June 2025
17th of July 2025	22nd of July 2025
15th of August 2025	20th of August 2025
17th of November 2025	20th of November 2025

Table 1: Maturity dates of the WTI Crude Oil futures option contracts and their respective underlying futures contract, traded on NYMEX on the 14th of March 2025.

η	H	ρ	a (fixed)
2.5238	0.0088	-0.8266	0.5

c_1	c_2	c_3	c_4	c_5	c_6
0.1223	0.1219	0.1260	0.1296	0.1494	0.1467

Table 2: Parameters of the model calibrated to the data from the 14th of March 2025.

smiles. We also plot the 95% confidence interval for the generated smile, as well as the trading volume, which is represented as a bar plot.

6 Conclusions and Future Research

Prior to this work, little to no information could be found in the literature on the applicability of rough volatility models in commodity markets. Our contribution to this area of research consists of a novel model to price futures options on commodities, in which the volatility process is given by the rBergomi model. We have calibrated it to real market data on WTI Crude Oil and obtained promising results.

Since this area remains largely unexplored, several directions for future research remain open. In particular, there is a need to develop an efficient calibration procedure to meet industry standards. It could also be interesting to investigate the usage of other rough volatility models, such as the rough Heston model, in the context of commodities.

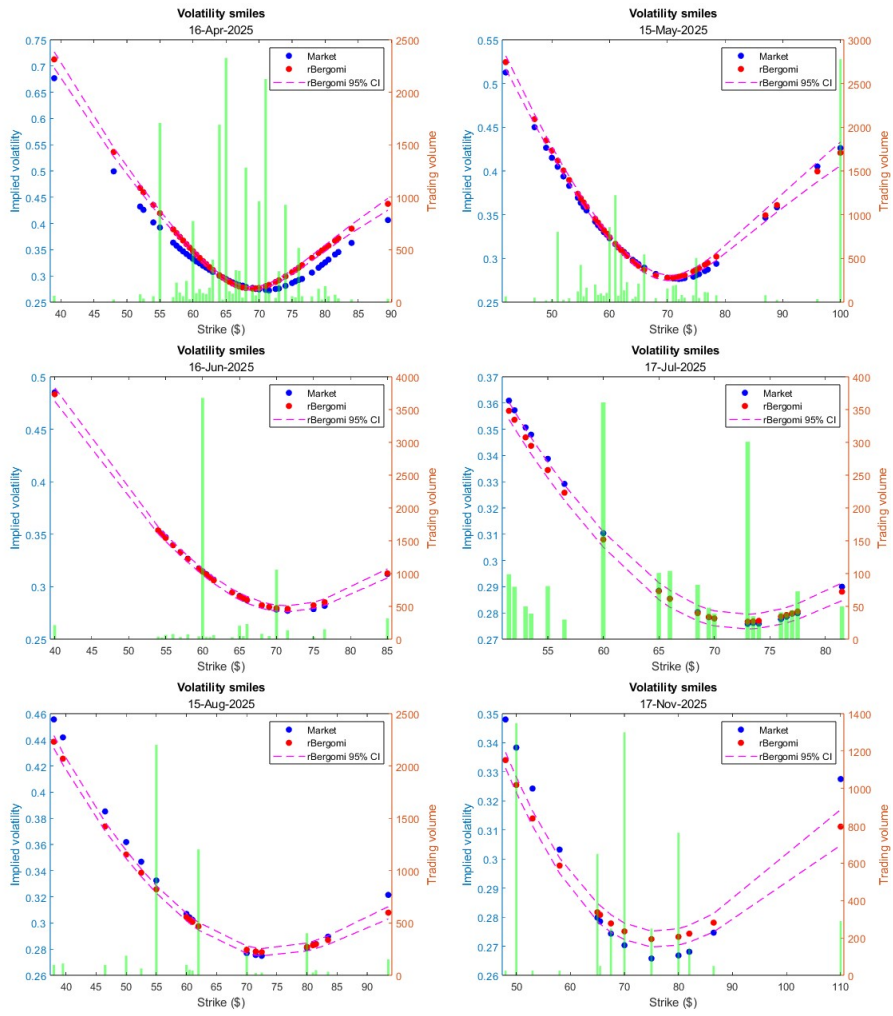


Figure 2: Calibration results for the data from the 14th of March 2025.

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